

Improved Method for Calculating Booster to Launch Pad Interface Transient Forces

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This paper presents a methodology for the calculation of the transient interface forces between a launch vehicle and its launch pad. The method presented is closed form and, thus, is mathematically exact. The method uses variable constraint equations to couple the launch vehicle to the pad. The resulting interface forces are monitored each time step, and as the launch vehicle separates from the pad, the constraint equations are modified accordingly. The only assumption in the method is that the forces acting on the booster and pad, including the subject interface forces, vary linearly between integration time steps. This is the same assumption used in standard linear transient loads analyses. In addition, the nature of the constraint equations allows a great reduction in the number of coupled equations to be solved. Solving the resulting small system of equations yields the interface forces, which are then used to solve for the booster and pad response by the standard linear transient solution method. Use of constraint equations also frees the analyst from calculating coupled booster-pad systems modes, which are a necessity in some methodologies currently in use.

Nomenclature‡

A, A', B, B'	= coefficients of integration
D	= all known variables to the modal displacement solution
F, F'	= coefficients of integration
f	= physical interface force
$\{f\}$	= physical force
G, G'	= coefficients of integration
I	= identity matrix
K	= discrete physical stiffness matrix
P	= generalized force
q	= dummy variable
V	= all known variables to the modal velocity solution
x	= discrete physical displacement
β	= modal damping coefficient
Δ	= displacement difference
δ	= interface displacement mismatch
ξ	= modal displacement degree of freedom
ϕ	= eigenvectors
ω	= eigenvalues
0	= null matrix
Subscripts	
b	= booster
i	= modal degree of freedom
j	= physical degree of freedom
k	= modal degree of freedom
n	= time step
p	= launch pad
Superscripts	
r	= residual mode
t	= untruncated modes, i.e., modes retained in the analysis
T	= matrix transpose

Introduction

ONE of the prime drivers in booster-payload system liftoff response is the time history of the pad forces. As the

launch vehicle's thrust increases, these forces decay rapidly. From an initial condition of static weight, winds, stacking tolerances, etc., these forces must come to zero as the booster separates from the launch pad. The sensitivity of the booster-payload system to these loads was demonstrated in Ref. 1. Reference 1 reports the results of a parametric study performed for Space Shuttle liftoff from the Vandenberg Air Force Base launch site. In the study, the launch mount stiffness characteristics are varied by including a set of springs between the booster and launch mount. The resulting impacts to the vehicle are reported. This study resulted in a specific set of springs being added to the launch mount design to serve as a load-alleviation device. These springs altered the pad force decay history, and the resulting changes in the response were used to reduce the liftoff loads' environment. This study demonstrated the sensitivity of the system loads to the booster-pad force history. Thus, it is important that booster-to-pad force histories be calculated as accurately as possible.

Linear Transient Solution

The standard linear solution to the booster-payload modal equations of motion is given by the following three equations:

$$\xi_{i,n+1} = F \xi_{i,n} + G \xi_{i,n} + A P_{i,n} + B P_{i,n+1} \quad (1)$$

$$\dot{\xi}_{i,n+1} = F' \xi_{i,n} + G' \xi_{i,n} + A' P_{i,n} + B' P_{i,n+1} \quad (2)$$

$$\ddot{\xi}_{i,n+1} = P_{i,n+1} - 2\beta \dot{\xi}_{i,n+1} - \omega_0^2 \xi_{i,n+1} \quad (3)$$

Equation (1) is the solution for the modal displacement, Eq. (2) for the modal velocity, and Eq. (3) for the modal acceleration.

In normal linear analyses, the generalized force input P is known. The right-hand sides of the equations are defined, making the solution for the $n+1$ time step a trivial task. However, for the case of booster-to-launch-pad separation, the pad forces acting on the booster may not be given. The three preceding equations are rewritten with the pad forces separated from all other forces.

$$\begin{aligned} \xi_{i,n+1} = & F \xi_{i,n} + G \xi_{i,n} + A P_{i,n} + B P_{i,n+1} \\ & + A \phi_{i,j}^T f_{j,n} + B \phi_{i,j}^T f_{j,n+1} \end{aligned} \quad (4)$$

$$\begin{aligned} \dot{\xi}_{i,n+1} = & F' \xi_{i,n} + G' \xi_{i,n} + A' P_{i,n} + B' P_{i,n+1} \\ & + A' \phi_{i,j}^T f_{j,n} + B' \phi_{i,j}^T f_{j,n+1} \end{aligned} \quad (5)$$

$$\ddot{\xi}_{i,n+1} = P_{i,n+1} - 2\beta \dot{\xi}_{i,n+1} - \omega_0^2 \xi_{i,n+1} + \phi_{i,j}^T f_{j,n+1} \quad (6)$$

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‡The nomenclature in this report follows that of the NASTRAN theoretical manual.²

The variable f is the physical pad forces acting on the booster, and ϕ is the booster eigenvector row partition containing the pad interface degrees of freedom. Note that the products $B\phi^T$ and $B'\phi^T$ are not typical matrix products. The B and B' coefficients of integration are actually scalar multipliers of P , i.e., $B_k P_k$. Thus, in the preceding equations, and those to follow, $B\phi^T$ denotes multiplication of row k of ϕ^T by the scalar B_k .

The pad forces are unknown for time $n+1$ and are typically solved by using some nonlinear analytic procedure. The method of this report will use constraint equations between booster and pad, thus keeping the pad forces as part of the solution. First, Eqs. (4-6) are rewritten in shorter form by placing all the known variables together.

$$\xi_{i,n+1} = D + B\phi_{i,j}^T f_{j,n+1} \quad (7)$$

$$\dot{\xi}_{i,n+1} = V + B'\phi_{i,j}^T f_{j,n+1} \quad (8)$$

$$\ddot{\xi}_{i,n+1} = P_{i,n+1} - 2\beta\dot{\xi}_{i,n+1} - \omega_0^2\xi_{i,n+1} + \phi_{i,j}^T f_{j,n+1} \quad (9)$$

Constraint Equations

Constraint equations will be used to enforce booster-to-launch-pad displacement compatibility. Assuming a zero mass launch pad, which is standard practice in Titan pad separation analyses, the launch pad displacements are given by the fol-

lowing equation:

$$[K_p]\{x_p\} = \{-f\} \quad (10)$$

This is the standard static equation of equilibrium, where K_p is the launch pad stiffness, x_p the launch pad physical displacements at the booster interface, and f the pad forces acting on the booster. The corresponding booster displacements are given by

$$\{x_b\} = [\phi]\{\xi\} \quad (11)$$

Displacement compatibility between the booster and the pad is enforced by using the following two equations:

$$\{x_p\} - \{x_b\} = \{\Delta x\} \quad (12)$$

$$\{\Delta x\} = \{0\} \quad (13)$$

The derivation assumes the booster and the pad to be in the same coordinate system, although the equations could be written to account for differences.

Coupled Equations of Motion

Writing the booster equations of motion in matrix form, including the launch pad and the booster displacements, enforcing displacement compatibility between the two, and bringing the unknown forces to the left-hand side of the equation yields the following matrix equation:

$$\begin{bmatrix} I & & & & & & & & & -B\phi^T \\ & I & & & & & & & & -B'\phi^T \\ & & \omega_0^2 & & & & & & & -\phi^T \\ & & & 2\beta & & & & & & \\ & & & & I & & & & & \\ & & & & & K_p & & & & I \\ & -\phi & & & & & I & & & \\ & & & & & & & -I & & \\ & & & & & & & & I & \\ & & & & & & & & & I \\ & & & & & & & & & & 0 \end{bmatrix} \begin{Bmatrix} \xi \\ \dots \\ \dot{\xi} \\ \dots \\ \ddot{\xi} \\ \dots \\ x_p \\ \dots \\ x_b \\ \dots \\ \Delta x \\ \dots \\ f \end{Bmatrix} = \begin{Bmatrix} D \\ \dots \\ V \\ \dots \\ P \\ \dots \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{Bmatrix} \quad (14)$$

Note that Eq. (14) has been assembled from Eqs. (7-13). The equation has the same form as multipoint equations used in many NASTRAN applications except that the degrees of freedom include both physical and modal, and also, the constraint forces f remain in the solution.

Reference 3 used constraint equations as part of the solution in transient solutions with static and kinetic friction. There the constraints were applied to the modified Newmark-Beta recursion formula. Here the constraints have been imposed on the closed-form modal solution. The advantage of both methods is that the unknown forces come directly from the solution. The advantage of the method is the mathematical exactness. No finite-difference approximations have been used.

The solution provided by Eq. (14) is exact and linear. It proceeds time step by time step, from the initial conditions, until one of the axial pad forces changes from compression to tension. The solution for this time step is invalid and is disregarded. The constraint equations are then modified, releasing the booster at the applicable degrees of freedom. The solution is then continued, time step by time step, until again the constraints between the booster and pad are modified. For the condition of the booster still in partial contact with the pad, Eq. (14) is modified and will appear as

$$\begin{bmatrix} I & & & & & & & & & -B\phi^T \\ & I & & & & & & & & -B'\phi^T \\ & & \omega_0^2 & & & & & & & -\phi^T \\ & & & 2\beta & & & & & & \\ & & & & I & & & & & \\ & & & & & K_p & & & & I \\ & -\phi & & & & & I & & & \\ & & & & & & & -I & & \\ & & & & & & & & I & \\ & & & & & & & & & I \\ & & & & & & & & & & 0 \\ & & & & & & & & 0 & I \\ & & & & & & & & & & I \\ & & & & & & & & & & & 0 \end{bmatrix} \begin{Bmatrix} \xi \\ \dots \\ \dot{\xi} \\ \dots \\ \ddot{\xi} \\ \dots \\ x_p \\ \dots \\ x_b \\ \dots \\ \Delta x \\ \dots \\ q \\ \dots \\ f \end{Bmatrix} = \begin{Bmatrix} D \\ \dots \\ V \\ \dots \\ P \\ \dots \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{Bmatrix} \quad (15)$$

Here the subscript b denotes the booster and subscript p denotes the launch pad. Note that P_p is the generalized external force, if any, acting on the pad, such as overpressure. Again, static condensation is used to minimize the computational effort. Going through the same two-step condensation performed earlier, yields

$$\begin{bmatrix} I & & & & \phi_p B_p \phi_p^T \\ & I & & & -\phi_b B_b \phi_b^T \\ & & I & & \\ & & & I & \\ & & & & 0 \end{bmatrix} \begin{Bmatrix} x_p \\ \dots \\ x_b \\ \Delta x \\ \dots \\ f \end{Bmatrix} = \begin{Bmatrix} \phi_p D_p \\ \dots \\ \phi_b D_b \\ 0 \\ \dots \\ 0 \end{Bmatrix} \quad (21)$$

The solution procedure follows the same steps as outlined earlier, i.e., the solution proceeds time step by time step, with the constraint equations being modified as required. Note that the number of degrees of freedom in Eq. (21) is only four times the number of interface degrees of freedom.

Stacking Misalignment Loads

The interface forces between the booster and launch pad typically include a set of static redundant forces. These forces are produced by such factors as stacking misalignments and cryogenic shrinkage. The method reported here can account easily for such forces and also include any dynamic effect associated with their decay during booster separation. This is accomplished by a simple modification to Eq. (13).

$$\{\Delta x\} = \{-\delta\} \quad (22)$$

Here δ is the displacement difference between the booster and launch pad due to the above-mentioned factors. This displacement mismatch is calculated with the booster uncoupled from the pad. Imposing Eq. (22) as a constraint on the system will then force the two components' interfaces together. The variable f will contain the necessary redundant forces. The advantage of this method is that, as the booster separates from the pad, a new set of redundant forces, based on the remaining coupled interface degrees of freedom, will be automatically included. Also, the decay of these redundant forces is included in the modal response. As the constraint equations are modified, the variable δ is adjusted by zeroing out the displacements for the degrees of freedom that have separated.

Including Truncated Flexibility

In standard closed-form modal analyses, the modal data are almost always truncated. When coupling components together, modal truncation errors at the interfaces must be taken into account. Various methods (see Ref. 4) have been developed to correct for a structural element's truncated flexibility, for a defined interface. These methods can be used to generate a set of residual modes, which are then used to correct for the truncation's artificial stiffening of the interface. These residual modes do not represent any real vibrational mode of the structure, but do accurately correct the systems' interface flexibility. Therefore, these modes should be used as a quasi-static effect only. This is the procedure used in Ref. 3.

The use of the constraint equations in the method presented here physically couples the booster to the pad. Therefore, any truncation errors at the interface degrees of freedom should be corrected. This will ensure two things: 1) that the redundant force calculations are accurate, and 2) that the coupled booster and pad system modal properties are represented accurately throughout the booster separation.

The displacement lost to modal truncation is given by use of the residual modes.

$$\{x^r\} = [\phi^r] \{\xi^r\} \quad (23)$$

The number of residual modes need only be equal to the number of degrees of freedom whose total flexibility is desired, in this case, the number of interface degrees of freedom. Since the residual modes are treated quasistatically, the modal displacement is given by

$$\{\xi^r\} = \left[\omega_0^{r-2} \right] \{P^r\} \quad (24)$$

The total physical displacement is the sum of the displacement calculated from the untruncated modes and the displacement from the residual modes.

$$\{x\} = \{x^t\} + \{x^r\} \quad (25)$$

In the preceding three equations, the superscript t denotes untruncated and superscript r denotes residual.

Incorporating these residual modes into Eq. (21), incorporating Eq. (22), and keeping booster-to-pad interface forces f , separate from the generalized force P , gives the following:

$$\begin{bmatrix} I & & & & \phi_p B_p \phi_p^T + \phi_p^r \omega_p^{r-2} \phi_p^{rT} \\ & I & & & -\phi_b B_b \phi_b^T - \phi_b^r \omega_b^{r-2} \phi_b^{rT} \\ & & I & & \\ & & & I & \\ & & & & 0 \end{bmatrix} \begin{Bmatrix} x_p \\ \dots \\ x_b \\ \Delta x \\ \dots \\ f \end{Bmatrix} = \begin{Bmatrix} \phi_p D_p + \phi_p^r \omega_p^{r-2} P_p^r \\ \dots \\ \phi_b D_b + \phi_b^r \omega_b^{r-2} P_b^r \\ 0 \\ \dots \\ -\delta \end{Bmatrix} \quad (26)$$

The treatment of the residual modes as quasistatic effects can be programmed easily by setting the coefficients of integration A , F , and G equal to zero. The residual modal velocities and accelerations are, by definition, always zero.

Note that the final equation for the solution of the pad interface forces f , could be even smaller than Eq. (26) by reducing out the x_p and x_b degrees of freedom. The resulting number of degrees of freedom would then become just twice the number of interface degrees of freedom! (For example, this is only 24 degrees of freedom for a Titan pad separation analysis.) This results in

$$\begin{bmatrix} I & & & \phi_p B_p \phi_p^T + \phi_p^r \omega_p^{r-2} \phi_p^{rT} + \phi_b B_b \phi_b^T + \phi_b^r \omega_b^{r-2} \phi_b^{rT} \\ & I & & \\ & & I & \\ & & & 0 \end{bmatrix} \begin{Bmatrix} \Delta x \\ \dots \\ f \end{Bmatrix} = \begin{Bmatrix} \phi_p D_p + \phi_p^r \omega_p^{r-2} P_p^r \\ -\phi_b D_b - \phi_b^r \omega_b^{r-2} P_b^r \\ \dots \\ -\delta \end{Bmatrix} \quad (27)$$

The constraint equations are controlled as described earlier, until the booster is completely free from the pad. The final

equation of motion is given by

$$\begin{bmatrix} I & \phi_p B_p \phi_p^T + \phi_p^r \omega_p^{r-2} \phi_p^{rT} \\ & + \phi_b B_b \phi_b^T + \phi_b^r \omega_b^{r-2} \phi_b^{rT} \\ 0 & I \end{bmatrix} \begin{Bmatrix} \Delta x \\ \dots \\ f \end{Bmatrix} = \begin{Bmatrix} \phi_p D_p + \phi_p^r \omega_p^{r-2} P_p^r \\ -\phi_b D_b - \phi_b^r \omega_b^{r-2} P_b^r \\ 0 \end{Bmatrix} \quad (28)$$

Thus, the closed-form solution for the pad interface forces requires the solution to a very small system of equations.

Summary of Solution Sequence

The algorithm to perform such an analysis would not go through all the preceding steps to get to Eq. (27), but would rather use Eq. (27) as its starting point. The coefficients of the equation are dependent on the modal parameters and the chosen integration time step. Starting with the initial conditions, the right-hand side of the equation is generated, the solution then yields the interface forces for the next time step. With these known, the modal solution is solved by use of the standard linear solution, i.e., Eqs. (7-9). The procedure is repeated time step by time step, the right-hand side being regenerated for each step. After the solution of each time step (and after the launch sequencer has mechanically broken the restraints between the pad and the booster), the pad interface forces are scrutinized. Any solution in which an axial pad force has gone into tension is invalid and is discarded. Equation (27) is then modified by changing the constraints accordingly. The solution sequence is then continued. The booster has lifted off of the pad completely when all constraints have been removed, and the equation appears as Eq. (28).

For some particular analyses, it may be desired to vary the time-step size. In a standard linear solution, this is easily accomplished. (The coefficients A , B , F , G , A' , B' , F' , and G' are simply recalculated using the new time step, denoted by h in Ref. 2.) However, when using the NASTRAN-supplied modified Newmark-Beta integration scheme, the analyst is locked into the chosen initial time step. A change in time-step size is a complicated affair for finite-difference integration procedures and is not even attempted in NASTRAN. Since the preceding procedure was derived from the linear solution, a change in time-step size is also accomplished easily. Note that this change would require regeneration of the left-hand side and a new solution (inversion) of this matrix. However, because of the very small size of the solution, this is a small expense.

Conclusions

Equation (27) is a very small system of coupled equations, the solution to which yields the booster-to-launch-pad interface forces. This system of equations was derived using the standard closed-form linear modal solution. It is, therefore, mathematically exact, the only assumption being that all forces vary linearly between the integration time steps. The equation accounts for the redundant forces generated by stacking misalignments, cryogenic shrinkage, etc. The equation has the necessary corrections for modal truncation, thus assuring both system modal accuracy and redundant force calculation accuracy. The sequence of booster separation from the pad is controlled in an exact manner by appropriate manipulation of the constraint equations. The solution to Eq. (27) is computationally very efficient. The small number of degrees of freedom ensures that any analyst, even those using

the smallest of modern computers, should have no problems caused by hardware limitation.

Appendix

The preceding derivation has been presented for the specific case of booster-payload separation from a launch pad. The same approach can be utilized to solve a large class of problems, both linear and nonlinear. The coupling of components by use of constraint equations as part of the equations of motion allows the analyst to use the modal properties of each element, without calculating the usual set of system modes. This has many advantages.

During spacecraft preliminary linear loads analyses, parametric studies on the spacecraft's structure may be desired. The preceding method allows the analyst to vary the spacecraft's structural characteristics without calculating the spacecraft-booster system modes for every iteration, and, at the same time, it results in a closed-form solution. Reference 5 presented a solution scheme for such problems, using a modified Newmark-Chan-Beta numerical integration scheme to solve for the transient response. The present approach does require an eigenvalue solution on the spacecraft, but it does avoid the use of finite-difference integration.

Another advantage of this approach deals with modal damping of the various components. Often each component has a different damping schedule. Standard practice within the aerospace industry transforms each component's damping into the system modal coordinate system. Damping from each element is then added together. This results in a system modal damping matrix that can be highly cross-coupled. In order to use the standard linear solution routines, all of the off-diagonal terms are stripped off, i.e., they are assumed to be zero. Using the preceding approach, the analyst can solve these transient problems, again closed form, without generating system modes, and with each structural element having its individual free-free modal diagonal damping characteristics.

This method is also directly applicable to analyses involving base driving. In these types of solutions, the constraint equations would consist of the base driven degrees of freedom being defined much as Δx was in Eq. (27). The corresponding time histories replace δ on the right-hand side. The system of equations could be base driven by displacement, acceleration, or even velocity time histories. Again, the solution is exact.

This method is readily adaptable to nonlinear problems. In fact, booster-pad separation analyses are considered to be nonlinear. However, the preceding derivation broke the problem down into a series of closed-form linear solutions. The same approach easily can be used to solve problems with gaps or dead bands. The analyst simply builds into the software, the logic to control the relative displacements, thus enforcing the displacement compatibility necessary to define the dead band.

The ability of the analyst to control constraints based on the system response would also simplify transient solutions involving impacts—a good example being a docking analysis that includes the flexible response. In such an analysis, the initial equation of motion appears similar to Eq. (28). The initial conditions consist of the rigid-body modal displacements and velocities. As the respective degrees of freedom around the docking rings come into contact, the constraint equations are modified. This continues until the docking rings are mated completely. The equation of motion then appears similar to Eq. (27), just the reverse sequence of pad separation. The transient analysis is then continued long enough for the desired response items to reach their maximums. Note that the response items could include any sensitive components on either space vehicle, such as solar arrays, antennas, etc.

The same approach, with or without constraint equations, could be used to solve transients problems with general nonlinearities, instead of using a finite-difference approach, e.g., the NASTRAN-provided Newmark-Beta solution routine. As in the Newmark-Beta method, the nonlinear forces would be

defined by the results of the previous time step. However, no finite-difference approximations are necessary. In fact, the analyst may desire the use of a predictor-corrector scheme to provide any desired degree of convergence to the nonlinear forces. Note that the scheme would have to iterate only on a small system of equations, similar to Eq. (28).

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